

# Corrected entropy-area relation and modified Friedmann equations

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ABSTRACT: Applying Clausius relation,  $\delta Q = TdS$ , to apparent horizon of a FRW universe with any spatial curvature, and assuming that the apparent horizon has temperature  $T = 1/(2\pi\tilde{r}_A)$ , and a quantum corrected entropy-area relation,  $S = A/4G + \alpha \ln A/4G$ , where  $\tilde{r}_A$  and  $A$  are the apparent horizon radius and area, respectively, and  $\alpha$  is a dimensionless constant, we derive modified Friedmann equations, which does not contain a bounce solution. On the other hand, loop quantum cosmology leads to a modified Friedmann equation  $H^2 = \frac{8\pi G}{3}\rho(1 - \rho/\rho_{\text{crit}})$ . We obtain an entropy expression of apparent horizon of FRW universe described by the modified Friedmann equation. In the limit of large horizon area, resulting entropy expression gives the above corrected entropy-area relation, however, the prefactor  $\alpha$  in the logarithmic term is positive, which seems not consistent with most of results in the literature that quantum geometry leads to a negative contribution to the area formula of black hole entropy.

KEYWORDS: Models of Quantum Gravity, Classical Theories of Gravity, Cosmology of Theories beyond the SM, Black Holes.

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## 1. Introduction

Due to the seminal work by Hawking [1], nowadays it is widely accepted that black hole behaves like a black body, emitting thermal radiation, with a temperature proportional to its surface gravity at the black hole horizon and with an entropy proportional to its horizon area [1, 2]. The Hawking temperature and horizon entropy together with the black hole mass obey the first law of black hole thermodynamics [3]. Hawking radiation is a quantum phenomenon. Therefore quantum theory, gravitational theory and statistical physics are connected each other in black hole thermodynamics. Einstein field equation holds in a way that the Hawking process satisfies the first law of thermodynamics. The development of black hole quantum physics and seeking for a self-consistent quantum theory of gravity lead people to consider the connection between the Einstein field equation and the first law of thermodynamics, although at the first glance, gravitational theory and thermodynamics have nothing to do with each other, because they belong to two completely different branches of science.

Assuming there is a proportionality between entropy and horizon area, Jacobson [4] derived the Einstein field equation by using the fundamental Clausius relation,  $\delta Q = TdS$ , connecting heat, temperature and entropy. The key idea is to demand that this relation holds for all the local Rindler causal horizon through each spacetime point, with  $\delta Q$  and  $T$  interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. In this way, Einstein field equation is nothing, but an equation of state of spacetime. Applying this idea to  $f(R)$  theory [5–7] and scalar-tensor theory [6, 8], it turns out that a nonequilibrium thermodynamic setup has to be employed. For another viewpoint, see [9, 10].

In the spirit of Jacobson’s derivation of Einstein field equation, one is able to derive Friedmann equations of a Friedmann-Robertson-Walker (FRW) universe with any spatial curvature by applying the Clausius relation to apparent horizon of the FRW universe [11]. This works not only in Einstein gravitational theory, but also in Gauss-Bonnet and Lovelock gravity theories. Here a key ingredient is to replace the entropy area formula in Einstein

theory by using entropy expressions of black hole horizon in those higher order curvature theories. For related discussions see also [12–14]. These results should closely relate to the fact that Einstein field equation can be rewritten as an unified first law [15–18]. Indeed, at the apparent horizon of FRW universe, the first Friedmann equation can be cast to a form [8, 19],  $dE_m = TdS + W_m dV$ . Here  $E_m = \rho V$  is the total energy inside of the apparent horizon with volume  $V$ ,  $W_m = (\rho - p)/2$  is the work density,  $T$  and  $S$  can be regarded as the temperature and entropy associated with the apparent horizon like a black hole horizon. This first law form also holds in RSII bran world scenario, warped DGP model and even more complicated case with a Gauss-Bonnet term in bulk [20–22]. Based on this, one is able to find the relation between entropy expression of apparent horizon and horizon geometry in bran world scenarios. These results have been summarized in [23]. For further discussions in this direction see [24–27]. On the other hand, there also exist some studies in the relation between Einstein field equation and first law of thermodynamics in the setup of black hole spacetime [28].

It is interesting to note that the Friedmann equations can be derived by using Clausius relation to the apparent horizon of FRW universe, in which entropy is assumed to be proportional to its horizon area. Also it is well-known that the so-called area formula of black hole entropy holds only in Einstein gravity. When some higher order curvature term appears in some gravity theory, the area formula has to be modified [29]. But it is clear from Wald formula that horizon entropy of black hole must be a function of horizon geometry. In this sense, it would be of great interest to see whether one is able to derive modified Friedmann equations if the entropy-area relation gets corrections by some reason. For example, a logarithmic term often occurs in literature

$$S = \frac{A}{4G} + \alpha \ln \frac{A}{4G}, \tag{1.1}$$

where  $A$  is the horizon area and  $\alpha$  is a dimensionless constant. Such a term appears in studying black hole entropy in loop quantum gravity (quantum geometry) [30–38], or in discussing the correction to black hole entropy due to thermal equilibrium fluctuation or quantum fluctuation [38–43]. However, even within the quantum geometry, the value of the prefactor  $\alpha$  is in debate. Some reference gives  $\alpha = -3/2$ , for example, [32]; some works lead to  $\alpha = -1/2$ , for example, [33–35]; the paper [36] argued that  $\alpha$  should be a positive integer and on the other hand, the author of [37] argued that  $\alpha$  should be equal to zero.

Applying the techniques of loop quantum gravity to homologous and isotropic space-time leads to the so-called loop quantum cosmology. Due to quantum correction, the Friedmann equations get modified. The big bang singularity is resolved and replaced by a quantum bounce [44]. For a brief summary on loop quantum cosmology, see [45]. Considering quantum correction, the modified Friedmann equation turns out to be (in the case of  $k = 0$ ) [45]

$$H^2 = \frac{8\pi G}{3} \rho \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right), \tag{1.2}$$

where  $\rho_{\text{crit}} = \sqrt{3}/(32\pi G^2 \gamma^3)$ ,  $\gamma$  is the so-called Barbero-Immirzi parameter. This parameter could be fixed as 0.2375 in order to give the area formula of black hole entropy in

loop quantum gravity [31]. Due to the corrected term in (1.2), the big bang singularity is replaced by a quantum bounce happening at  $\rho = \rho_{\text{crit}}$ .

The aim of this paper is twofold. The first is to derive modified Friedmann equations by applying Clausius relation to the apparent horizon of FRW universe and assuming the horizon has an entropy expression like (1.1) or a more general form. The other is to see whether the corrected entropy-area relation from loop quantum gravity will lead to the modified Friedmann equation (1.2) in loop quantum cosmology because the approaches to reach these two results seem different, although both of them are in the field of loop quantum gravity.

The organization of this paper is as follows. In the next section we obtain the modified Friedmann equations starting from the corrected entropy-area relation (1.1) by use of Clausius relation to the apparent horizon of FRW universe. In section 3 we get the entropy expression associated with apparent horizon in FRW universe described by the modified Friedmann equation (1.2) in loop quantum cosmology. The conclusion is included in section 4.

## 2. From corrected entropy-area relation to modified Friedmann equations

To make the paper be self-contained, let us start the case [11] without the corrected term in the entropy-area relation (1.1). The FRW universe is described by metric

$$ds^2 = -dt^2 + a^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right) = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega_2^2, \quad (2.1)$$

where  $x^0 = t$ ,  $x^1 = r$ ,  $h_{ab} = \text{diag}(-1, a^2/(1 - kr^2))$ ,  $\tilde{r} = a(t)r$  and  $k$  denotes the spatial curvature. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation  $h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0$ . With this, it is easy to find out the radius of the apparent horizon

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \quad (2.2)$$

where  $H = \dot{a}/a$  is the Hubble parameter and overdot stands for the derivative with respect to cosmic time  $t$ .

Suppose that the energy-momentum tensor  $T_{\mu\nu}$  of the matter in the universe has the form of a perfect fluid  $T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}$ , where  $\rho$  and  $p$  are the energy density and pressure, respectively. The energy conservation law then leads to the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (2.3)$$

Following [17], we define the work density  $W$  and energy-supply vector  $\Psi$  as

$$W = -\frac{1}{2}T^{ab}h_{ab}, \quad \Psi_a = T_a^b \partial_b \tilde{r} + W \partial_a \tilde{r}, \quad (2.4)$$

where  $T_{ab}$  is the projection of the (3 + 1)-dimensional energy-momentum tensor  $T_{\mu\nu}$  of matter in the FRW universe in the normal direction of 2-sphere. In our case, these are

$$W = \frac{1}{2}(\rho - p), \quad \Psi_a = -\frac{1}{2}(\rho + p)H\tilde{r}dt + \frac{1}{2}(\rho + p)adr. \quad (2.5)$$

With this, we can compute the amount of energy crossing the apparent horizon during the time interval  $dt$  [11]

$$\delta Q = -A\Psi = A(\rho + p)H\tilde{r}_A dt, \tag{2.6}$$

where  $A = 4\pi\tilde{r}_A^2$  is the area of the apparent horizon. To proceed, we make two assumptions: one is that the apparent horizon has an horizon area entropy like black hole horizon; the other is that the apparent horizon has a temperature, they have following forms [11]

$$S = \frac{A}{4G}, \quad T = \frac{1}{2\pi\tilde{r}_A}. \tag{2.7}$$

Then using the Clausius relation  $\delta Q = TdS$ , we can reach

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p). \tag{2.8}$$

Note that here we have used the relation

$$\dot{\tilde{r}}_A = -H\tilde{r}_A^3 \left( \dot{H} - \frac{k}{a^2} \right), \tag{2.9}$$

which comes directly from (2.2). Furthermore, using the continuity equation (2.3) and integrating (2.8), we can obtain

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \tag{2.10}$$

where an integration constant, which is just the cosmological constant, has been absorbed into the energy density  $\rho$ . Eqs. (2.8) and (2.10) are nothing, but the Friedmann equations of FRW universe. They are, of course, the concrete forms of Einstein field equations,  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}$ , in the FRW metric (2.1).

Thus, we derive the Friedmann equations of a FRW universe with any spatial curvature by applying the Clausius relation to apparent horizon of the FRW universe and assuming that the apparent horizon has an entropy satisfying the area formula like black hole horizon and a temperature  $T = 1/2\pi\tilde{r}_A$ .

Now we will apply the same idea to derive corresponding modified Friedmann equations to the corrected entropy-area relation (1.1), although we did not yet know the modified Einstein field equations due to quantum correction, which produces the corrected entropy-area relation (1.1). To go on, there are two key points, where seem worth mentioning here. Since we are still considering a FRW universe, the assumption keeps unchanged, that the apparent horizon has a temperature  $T = 1/(2\pi\tilde{r}_A)$ , because we know that Hawking temperature of black hole is completely determined by spacetime metric (geometry), independently of gravity theories, in which the black hole solution exists, while black hole horizon entropy depends on gravity theories under consideration. The other point is that since we are considering the perfect fluid matter as source in the universe, in that case, the amount of energy crossing the apparent horizon during the time interval  $dt$  is still given by (2.6). That is to say, in this case, the only difference is to replace area entropy  $S = A/4G$  by the corrected entropy-area relation (1.1).

Thus the Clausius relation,  $\delta Q = TdS$ , in this time leads to

$$A(\rho + p)H\tilde{r}_A dt = \frac{1}{2\pi\tilde{r}_A} \left( \frac{1}{4G} + \frac{\alpha}{A} \right) dA \quad (2.11)$$

With the help of (2.9), the above equation can be changed into

$$\left( 1 + \frac{4G\alpha}{A} \right) \left( \dot{H} - \frac{k}{a^2} \right) = -4\pi G(\rho + p). \quad (2.12)$$

Using the continuity equation, we can further rewrite (2.12) as

$$\left( 1 + \frac{\alpha G}{\pi} \left( H^2 + \frac{k}{a^2} \right) \right) \frac{d(H^2 + k/a^2)}{dt} = \frac{8\pi G}{3} \dot{\rho}. \quad (2.13)$$

Integrating (2.13) yields

$$H^2 + \frac{k}{a^2} + \frac{\alpha G}{2\pi} \left( H^2 + \frac{k}{a^2} \right)^2 = \frac{8\pi G}{3} \rho, \quad (2.14)$$

where an integration constant has been absorbed into the energy density, again. Eqs. (2.12) and (2.14) are nothing, but the corresponding modified Friedmann equations to the corrected entropy-area relation (1.1).

Note that (2.14) can be further rewritten as

$$H^2 + \frac{k}{a^2} = \frac{\pi}{\alpha G} \left( -1 + \sqrt{1 + \frac{16\alpha G^2}{3} \rho} \right). \quad (2.15)$$

If  $\alpha$  is viewed as a small quantity, expanding the right hand side of (2.15), up to the linear order of  $\alpha$ , we have

$$H^2 + \frac{k}{a^2} \approx \frac{8\pi G}{3} \rho \left( 1 - \frac{4\alpha G^2}{3} \rho \right). \quad (2.16)$$

Let us mention here that eq. (2.14) has another positive root for  $(H^2 + \frac{k}{a^2})$ , but that solution has no classical limit as  $\alpha \rightarrow 0$ . We note that (2.16) is quite similar to the modified Friedmann equation (1.2) in loop quantum cosmology. We see from (2.16) that if  $\alpha > 0$ , there seemly exists a bounce at

$$\rho = \tilde{\rho}_{\text{crit}} = \frac{3}{4\alpha G^2}. \quad (2.17)$$

but this is not true. It is because (2.16) is an approximate solution, it holds only as  $\rho/\tilde{\rho}_{\text{crit}} \ll 1$ . In fact, it is easy to see from (2.14) that there does not exist any bounce solution in (2.14). Notice the fact that eq. (1.1) is obtained in the limit of large horizon area, it is not surprised that the corrected entropy-area relation does not lead to a modified Friedmann equation with a bounce solution because the latter is a nonperturbative effect.

When  $\alpha < 0$ , it is easy to see from (2.14) that in this case, there exists a de Sitter solution as the late-time attractor of (2.14):  $H^2 + k/a^2 = 2\pi/(|\alpha|G)$ .

In fact, the above procedure to derive the modified Friedmann equations by using the corrected entropy-area relation can be further generalized. For example, the next quantum

correction to black hole entropy (1.1) is widely believed to have the form  $4G/A$ , which means [46]

$$S = \frac{A}{4G} + \alpha \ln \frac{A}{4G} + \beta \frac{4G}{A}, \quad (2.18)$$

where  $\beta$  is another dimensionless constant. By using the same procedure, it is easy to show that in this case, the modified Friedmann equations are

$$\left(1 + \alpha \frac{4G}{A} - \beta \frac{16G^2}{A^2}\right) \left(\dot{H} - \frac{k}{a^2}\right) = -4\pi G(\rho + p), \quad (2.19)$$

$$H^2 + \frac{k}{a^2} + \frac{\alpha G}{2\pi} \left(H^2 + \frac{k}{a^2}\right)^2 - \frac{\beta G^2}{3\pi^2} \left(H^2 + \frac{k}{a^2}\right)^3 = \frac{8\pi G}{3} \rho, \quad (2.20)$$

where  $A = 4\pi/(H^2 + k/a^2)$  is the area of apparent horizon. Finally we stress that the above procedure also works well for a more general case: the entropy is a function of horizon geometry. For example, suppose the apparent horizon has an entropy with form  $f(A/4G)$ , that is,

$$S = f(x), \quad x = \frac{A}{4G}, \quad (2.21)$$

where  $f$  is an arbitrary function of horizon area. Applying the Clausius relation  $\delta Q = TdS$  to the apparent horizon of FRW universe, we can obtain the second Friedmann equation concerning the derivative of Hubble parameter

$$\left(\dot{H} - \frac{k}{a^2}\right) f'(x) = -4\pi G(\rho + p), \quad (2.22)$$

where a prime stands for the derivative with respect to  $x$ . Using the continuity equation, we can reach

$$\frac{8\pi G}{3} \rho = -\frac{\pi}{G} \int \frac{f'}{x^2} dx. \quad (2.23)$$

This is nothing, but the first Friedmann equation corresponding to the apparent horizon having the entropy form (2.21).

### 3. From modified Friedmann equations to corrected entropy-area relation

In this section, we will derive an entropy expression associated with apparent horizon of a FRW universe described by the modified Friedmann equation (1.2) by using the method proposed in [20]. The starting point is the unified first law [15–17]: The Einstein field equation can be rewritten as

$$dE = A\Psi + WdV, \quad (3.1)$$

where  $A = 4\pi\tilde{r}^2$  and  $V = \frac{4\pi}{3}\tilde{r}^3$  are area and volume of 3-dimensional sphere with radius  $\tilde{r}$ ,  $E$  is the Misner-Sharp energy

$$E = \frac{\tilde{r}}{2G} (1 - h^{ab} \partial_a \tilde{r} \partial_b \tilde{r}), \quad (3.2)$$

$\Psi$  and  $W$  are defined by (2.4). According to thermodynamics, entropy is associated with heat flow as  $\delta Q = TdS$ , and heat flow is related to change of energy of a given system. As

a consequence, entropy is finally associated with the energy-supply term. The latter can be rewritten as

$$A\Psi = \frac{\kappa}{8\pi G}dA + \tilde{r}d(E/\tilde{r}), \quad (3.3)$$

where  $\kappa$  is the surface gravity defined as

$$\kappa = \frac{1}{2\sqrt{-h}}\partial_a(\sqrt{-h}h^{ab}\partial_b\tilde{r}). \quad (3.4)$$

On the apparent horizon, the last term in (3.3) vanishes, then one can assign an entropy  $S = A/4G = \pi\tilde{r}_A^2/G$  to the apparent horizon with radius  $\tilde{r}_A$ . That is, in Einstein gravity, apparent horizon can be assigned an entropy proportional to its area like black hole entropy. After projecting along the vector  $\xi = \partial_t - (1 - 2\epsilon)Hr\partial_r$  with  $\epsilon = \dot{\tilde{r}}_A/2H\tilde{r}_A$ , we can get the first law of thermodynamics of apparent horizon [20]

$$\langle dE, \xi \rangle = \frac{\kappa}{8\pi G}\langle dA, \xi \rangle + \langle WdV, \xi \rangle, \quad (3.5)$$

where  $\kappa = -(1 - \epsilon)/\tilde{r}_A$  by definition. Next we can obtain the entropy expression associated with apparent horizon by using Clausius relation,  $\delta Q = TdS$ , here the heat flow  $\delta Q$  should be given by pure matter energy-supply  $A\Psi_m$ . To get pure matter energy-supply vector  $\Psi_m$ , we could rewrite (1.2) as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho + \rho_e), \quad (3.6)$$

where we have added a curvature term  $k/a^2$ , and  $\rho_e = -\rho^2/\rho_{\text{crit}}$ . Note that whatever the value of  $k$  is, the following conclusion is unchanged. Using continuity equation, we can obtain the effective pressure corresponding to the effective energy density,  $p_e = -\rho(\rho + 2p)/\rho_{\text{crit}}$ . Then we have associated energy-supply vector  $\Psi_e$  and work density  $W_e$

$$\begin{aligned} \Psi_e &= \frac{\rho}{\rho_{\text{crit}}}(\rho + p)H\tilde{r}dt - \frac{\rho}{\rho_{\text{crit}}}(\rho + p)adr, \\ W_e &= \frac{\rho}{\rho_{\text{crit}}}p. \end{aligned} \quad (3.7)$$

We have from (3.5) that the heat flow of pure matter

$$\begin{aligned} \delta Q &\equiv \langle A\Psi_m, \xi \rangle = \frac{\kappa}{8\pi G}\langle dA, \xi \rangle - \langle A\Psi_e, \xi \rangle \\ &= -\frac{HA\epsilon(1 - \epsilon)}{2\pi G}\frac{1}{\sqrt{\tilde{r}_A^2 - \frac{3}{2\pi G\rho_{\text{crit}}}}} \\ &= T\left\langle \frac{2\pi\tilde{r}_A^2}{G}\frac{d\tilde{r}_A}{\sqrt{\tilde{r}_A^2 - \frac{3}{2\pi G\rho_{\text{crit}}}}}, \xi \right\rangle \\ &= T\langle dS, \xi \rangle \end{aligned} \quad (3.8)$$

where we have defined  $T = \kappa/2\pi$ . Integrating (3.8), we reach

$$S = \frac{\pi\tilde{r}_A}{G}\sqrt{\tilde{r}_A^2 - \frac{3}{2\pi G\rho_{\text{crit}}}} + \frac{3}{2\pi G^2\rho_{\text{crit}}}\ln\left(\tilde{r}_A + \sqrt{\tilde{r}_A^2 - \frac{3}{2\pi G\rho_{\text{crit}}}}\right) + C, \quad (3.9)$$



where  $C$  is a constant, whose value should be determined by some physical condition. It is easy to show that with the entropy (3.9), the first law of thermodynamics for apparent horizon holds [19]

$$dE_m = TdS + W_m dV, \tag{3.10}$$

where  $E_m = \rho V$ ,  $W_m = (\rho - p)/2$  and  $V = 4\pi\tilde{r}_A^3/3$  are total energy inside the apparent horizon, work density and volume of the apparent horizon, respectively. In addition, let us notice that in the limit of large apparent horizon, entropy (3.9) has the form

$$S = \frac{A}{4G} + \frac{3}{4\pi G^2 \rho_{\text{crit}}} \ln \frac{A}{4G} + o\left(\frac{1}{A}\right) + C_0. \tag{3.11}$$

where  $A = 4\pi\tilde{r}_A^2$  is the area of apparent horizon,  $C_0$  is another constant and  $3/4\pi G^2 \rho_{\text{crit}} = 8\sqrt{3}\gamma^3 \approx 0.1856$ . In this case, entropy expression (3.11) has the same form as (1.1). But here the prefactor in the logarithmic term  $\alpha' = 3/4\pi G^2 \rho_{\text{crit}}$  is positive. Note that (1.1) is given in the limit of large horizon area. Therefore although the form of our result (3.11) is the same as (1.1), the corrected term gives an opposite contribution to the area entropy as the one quantum geometry does.

#### 4. Conclusions

In summary, applying the Clausius relation  $\delta Q = TdS$  to apparent horizon of a FRW universe with any spatial curvature and assuming that the apparent horizon has temperature  $T = 1/(2\pi\tilde{r}_A)$ , and an entropy obeying the well-known one quarter horizon area formula like black hole horizon, one is able to derive Friedmann equations governing the dynamical evolution of the universe. We followed the same idea to derive modified Friedmann equations from a quantum corrected entropy-area relation (1.1). However, resulting modified Friedmann equations do not contain a bounce solution. This result is not in contradiction with (1.2) since (1.1) holds only in the limit of large horizon area. We also derived corresponding modified Friedmann equations in the case with the apparent horizon having an entropy which is an arbitrary function of horizon area.

On the other hand, starting from the modified Friedmann equation (1.2) in loop quantum cosmology, we obtained an entropy expression associated with the apparent horizon of a FRW universe by use of the unified first law for dynamic horizon. In the limit of large horizon radius, resulting entropy indeed has the same form as the corrected entropy-area relation (1.1). However, the prefactor in the logarithmic term is positive, which seems in contradiction with most of results in literature that quantum geometry gives a negative contribution to the area formula of black hole entropy.

Finally, let us mention that although we derived modified Friedmann equations corresponding to the corrected entropy-area relation (1.1) by using Clausius relation, it would be of great interest to see whether one is able to get modified Einstein field equation by following Jacobson [4]. If this works, it further shows that given a thermodynamical relation between entropy and geometry, one is able to derive corresponding modified Einstein field equation, showing an interesting connection between them.

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